The problem of meeting of N of fuzzy objects

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1. Introduction

The first research of the differential equations with set-valued right-hand side has been fulfilled by A. Marchaud [1,2] and S.C. Zaremba [3]. In the early sixties, T. Wazewski [4,5], A.F. Filippov [6] had been obtained fundamental results about existence and properties of solutions of the differential equations with set-valued right-hand side (differential inclusions). Connection deriving between differential inclusions and optimum control problems was one of the most important outcomes of these papers. These outcomes became impulse for development of the theory of differential inclusions [7-9].

Considering of the differential inclusions required to study properties of set-valued maps, i.e. an elaboration the whole tool of mathematical analysis for set-valued maps [7,10,11].

In work [12] annotate of an R-solution for differential inclusion is introduced as an absolutely continuous set-valued maps. Various problems for the R-solution theory were considered in [8,13]. The basic idea for a development of an equation for R-solutions (integral funnels) is contained in [14].

In the eighties the last century the control theory in the conditions of uncertainty began to be formed. The control differential equations with set of initial conditions [15-17], control set differential equations [18-21] and the control differential inclusions [21-32] are used in the given theory for exposition of dynamic processes.

In recent years, the fuzzy set theory introduced by Zadeh [33] has emerged as an interesting and fascinating branch of pure and applied sciences. The applications of fuzzy set theory can be found in many branches of regional, physical, mathematical, differential equations, and engineering sciences. Recently there have been new advances in the theory of fuzzy differential equations [34-47] and inclusions [48-53] as well as in the theory of control fuzzy differential equations [54-57] and inclusions [57-59].

In this talk we consider a problem of a meeting of fuzzy linear objects and we receive a necessary condition of an optimality.

2. The fundamental definitions and designations

Let $comp(R^n)$ $(conv(R^n))$ be a set of all nonempty (convex) compact subsets from the space R^n,

$$h(A,B)=\min_{r\geq 0}\{S_r(A)\supset B,\ S_r(B)\supset A\}$$

be Hausdorff distance between sets A and B, $S_r(A)$ is r-neighborhood of set A.

Let E^n be the set of all $u: R^n\to[0,1]$ such that u satisfies the following conditions:

- u is normal, that is, there exists an $x_0\in R^n$ such that $u(x_0)=1$;
- u is fuzzy convex, that is, $u(\lambda x+(1-\lambda)y)\geq\min\{u(x),u(y)\}$ for any $x,y\in R^n$ and $0\leq\lambda\leq 1$;
- u is upper semicontinuous,
- $[u]^0=cl\{x\in R^n: u(x)>0\}$ is compact.

If $u\in E^n$, then u is called a fuzzy number, and E^n is said to be a fuzzy number space. For $0<\alpha\leq 1$, denote $[u]^\alpha=\{x\in R^n: u(x)\geq\alpha\}$.

Then from 1)-4), it follows that the α-level set $[u]^\alpha\in conv(R^n)$ for all $0\leq\alpha\leq 1$.

Let θ be the fuzzy mapping defined by $\theta(x)=0$ if $x\neq 0$ and $\theta(0)=1$.

Define $D : E^n \times E^n \rightarrow [0, \infty)$ by the relation

$$D(u, v) = \sup_{0 \le \alpha \le 1} h([u]^\alpha, [v]^\alpha),$$

where h is the Hausdorff metric defined in $comp(R^n)$. Then D is a metric in E^n.

Further we know that [60]:

1. (E^n, D) is a complete metric space,

2. $D(u + w, v + w) = D(u, v)$ for all $u, v, w \in E^n$,

3. $D(\lambda u, \lambda v) = |\lambda| D(u, v)$ for all $u, v \in E^n$ and $\lambda \in R$.

Now we consider following control differential equations with the fuzzy parameter

$$\dot{x} = f\left(t, x, w, v\right), \quad x(0) = x_0, \tag{1}$$

where $t \in R$ is the time; $x \in R_n$ is the state; $w \in R_m$ is the control; $v \in V \in E_k$ is the fuzzy parameter; $f : R_+ \times R_n \times R_m \times R_k \to R_n$.

Let $W : R_+ \to conv(R_m)$ be the measurable set-valued map.

Definition 5. The set LW of all measurable single-valued branches of the set-valued map $W\left(\cdot\right)$ is the set of the admissible controls.

Further we consider following control fuzzy differential inclusions

$$\dot{x} \in F(t, x, w), \quad x(0) = x_0, \tag{2}$$

where $F : R_+ \times R^n \times R^m \to E^n$ is the fuzzy map such that $F(t, x, w) \equiv f(t, x, w, V)$.

Obviously, the control fuzzy differential inclusion (2) turns into the ordinary fuzzy differential inclusion

$$\dot{x} \in \Phi(t, x), \quad x(0) = x_0, \tag{3}$$

if the control $\bar{w}(\cdot) \in LW$ is fixed and $\Phi(t, x) \equiv F(t, x, \bar{w}(t))$.

If right-hand side of the fuzzy differential inclusion (3) satisfies some conditions (for example look [12]) then the fuzzy differential inclusions (3) has the fuzzy R-solution.

Let $X(t)$ denotes the fuzzy R-solution of the differential inclusion (3), then $X(t, w)$ denotes the fuzzy R-solution of the control differential inclusion (2) for the fixed $w(\cdot) \in LW$.

Definition 6. The set $Y(T) = \{X(T, w) : w(\cdot) \in LW\}$ be called the attainable set of the fuzzy system (2).

3. The some properties of the fuzzy R-solution and time-optimal problem

3.1. The some properties of the fuzzy R-solution

Further in the given paper, we consider following control linear fuzzy differential inclusions

$$\dot{x} \in A(t)x + G(t, w),\ x(0) = x_0, \tag{4}$$

where $A(t)$ is $(n \times n)$ -dimensional matrix-valued function; $G : R_+ \times R^m \to E^n$ is the fuzzy map.

In this section, we consider the some properties of the fuzzy R-solution of the control fuzzy differential inclusion (4).

Let the following condition is true.

Condition A:

1) $A(\cdot)$ is measurable on $[0,T]$;

2) The norm $\|A(t)\|$ of the matrix $A(t)$ is integrable on $[0,T]$;

3) The set-valued map $W : [0,T] \to conv(R^m)$ is measurable on $[0,T]$;

4) The fuzzy map $G : [0,T] \times R^m \to E^n$ satisfies the conditions

a) measurable in t; b) continuous in w;

5) There exist $v(\cdot) \in L_2[0,T]$ and $l(\cdot) \in L_2[0,T]$ such that $h(W(t),0) \le v(t)$, $D(G(t,w),\theta) \le l(t)$

almost everywhere on $[0,T]$ and all $w \in W(t)$.

6) The set $Q(t) = \{G(t, w(t)) : w(\cdot) \in LW\}$ is compact and convex for almost every $t \in [0,T]$, i.e. $Q(t) \in conv(E_n)$.

Theorem 1 [61]. Let the condition A is true.

Then for every $w(\cdot) \in LW$ there exists the fuzzy R-solution $X(\cdot, w)$ such that

1) the fuzzy map $X(\cdot, w)$ has form

$$X(t, w) = \Phi(t)\, x_0 + \Phi(t) \int_0^t \Phi^{-1}(s) G(s, w(s)) ds,$$

where $t \in [0,T]$; $\Phi(t)$ is Cauchy matrix of the differential equation $\dot{x} = A(t)x$;

2) $X(t, w) \in E^n$ for every $t \in [0,T]$;

3) the fuzzy map $X(\cdot, w)$ is the absolutely continuous fuzzy map on $[0,T]$.

Theorem 2 [61]. Let the condition A is true. Then the attainable set $Y(T)$ is compact and convex.

Remark. Properties of space $comp(E^n)$ have been considered in work [62].

3.2. Time-optimal problem

Consider the following time-optimal problem: it is necessary to find the minimal time T and the control $w_*(\cdot) \in LW$ such that the fuzzy R-solution of system (4) satisfy of the condition:

$$X\left(T, w_*\right) \cap S_k \neq \varnothing, \tag{5}$$

where $S_k \in E^n$ is the fuzzy terminal set.

Theorem 3 [61]. (necessary optimal condition for the time-optimal problem (4),(5)). Let the condition A is true and the pair $(T, w_*(\cdot))$ is optimality of the control problem (4),(5).

Then there exists the vector-function $\psi(\cdot)$, which is the solution of the system $\dot{\psi} = -A^T(t)\psi$, $\psi(T) \in S_1(0)$ such that

1) $C([G(t, w_*)]_1, \psi(t)) = \max\limits_{w \in W(t)} C([G(t, w)]_1, \psi(t))$

almost everywhere on $\left[0,\mathrm{T}\right]$;

2) $C([X(T, \overset{*}{w})]^1, \psi(T)) = -C^1([S_k], -\psi(T))$.

4. The problem of meeting of N of fuzzy objects

Further, we consider N linear control differential inclusions with fuzzy parameters

$$\dot{x}_i \in A_i(t)\, x_i + G_i(t, w_i), \quad x_i(0) = x_0^i,\ i = \overline{1, N}, \tag{6}$$

where $x_i \in R_n$; $t \in R_+$; $A_i(t) : R_+ \to R_{n\times n}$ is a matrix $n \times n$; $G_i(t, w_i) : R_+ \times R_{k_i} \to E_n$ is a fuzzy map; $w_i \in W_i \subset R_{k_i}$ is a control parameter; $x_0^i \in R_n$.

Consider the following optimal control problem (<u>problem ***A***</u>***):*** it is necessary to find the minimal time T_* and controls $w_*^i(\cdot) \in LW_i$, $i = \overline{1, N}$ such that the fuzzy R-solutions of system (6) satisfy of the condition:

$$X_1(T^*, w_*^1) \cap ... \cap X_N(T^*, w_{*N}) \neq \varnothing . \tag{7}$$

Definition 6. The collection $\left(T_*, w_*^1(\cdot), ..., w_{*N}(\cdot)\right)$ is said to be optimality if

$$X_1(T^*, w_*^1) \cap ... \cap X_N(T^*, w_{*N}) \neq \varnothing \text{ and } \quad X_1(\tau, w_1) \cap ... \cap X_N(\tau, w_N) = \varnothing$$

for every $0 \le \tau < T_*$ and all $w_i(\cdot) \in LW_i$, $i = \overline{1, N}$.

Further we will reduce necessary conditions of an optimality of collection $(T_*, w_*^1(\cdot), ..., w_{*N}(\cdot))$ for meeting problem.

Theorem 4. Let the following conditions hold for every $i \in \{1,\ldots,N\}$:

1) $A_i(\cdot)$ is measurable on $[0,T^*]$;

2) The norm $\|A_i(t)\|$ of the matrix $A_i(t)$ is integrable on $[0,T^*]$;

3) The set-valued map $W_i:[0,T^*] \to conv(R^{k_i})$ is measurable on $[0,T^*]$;

4) The fuzzy map $G_i:[0,T^*] \times R^{k_i} \to E^n$ satisfies the conditions

a) measurable in t; b) continuous in w_i;

5) There exist $v_i(\cdot) \in L_2[0,T^*]$ and $l_i(\cdot) \in L_2[0,T^*]$ such that

$$h(W_i(t),0) \le v_i(t),\ \ D(G_i(t, w_i),\theta) \le l_i(t)$$

almost everywhere on $[0,T^*]$ and all $w_i \in W_i(t)$.

6) The set $Q_i(t) = \{G_i(t, w_i(t)) : w_i(\cdot) \in LW_i\}$ is compact and convex for almost every $t \in [0,T^*]$, i.e. $Q_i(t) \in conv(E^n)$

and the pair $(T_*, w_*^1(\cdot), \ldots, w_*^N(\cdot))$ is optimality for the problem (6),(7).

Then there exist $j \in \{1,\dots,N\}$ and solution $\psi^j(\cdot)$ of the differential equation $\dot{\psi}_j = -\left(A_j(t)\right)^T \psi_j$, $\|\psi_j(T^*)\| = 1$ such that

1) $C([G_j(t, w_*^j)]_1, \psi_j(t))_j = \max\limits_{w^j \in W(t)} C([G_j(t, w_j)]_1, \psi_j(t))$

almost everywhere on $[0, T^*]$;

2) $C([X^j(T^*, w_*^j)]^1, \psi^j(T^*)) = -C([\sum\limits_{i=1}^{N} X_i\left(T^*, w_*^i\right)]_1, -\psi_j(T^*))$.

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